Performance Assessment Task

Piles of Oranges
Grade 4

The task challenges a student to demonstrate understanding of the concepts involved in generating and analyzing a pattern. Students must be able to extend a given pattern and corresponding numeric table. A student must determine the result of the rule for a specific future value. A student must understand and use inverse operations and the strategy of working backwards to make an argument for the incorrectness of a given mathematical statement.

Common Core State Standards Math - Content Standards

Operations and Algebraic Thinking

Generate and analyze patterns.

4.OA.5 Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself.

Common Core State Standards Math – Standards of Mathematical Practice

MP.1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MP.3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Assessment Results

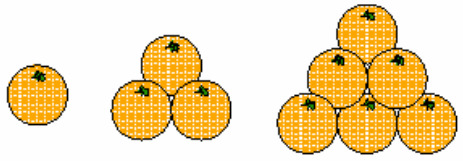
This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

Grade Level	Year	Total Points	Core Points	% At Standard
4	2004	8	4	59 %

Piles of Oranges

This problem gives you the chance to:
• describe, extend and make generalizations about a number pattern

Here are some piles of oranges that are displayed in Mrs. Chang's grocery store.



	pile # 1	pile # 2	pile # 3	pile # 4
number of oranges	1	1 + 2 = 3	1 + 2 + 3 = 6	

1. Draw pile # 4 of oranges next to pile # 3 in the diagram above.

2. How many oranges are needed for pile # 4 and pile # 5?
   Write your answers in the table below.

Pile #	1	2	3	4	5
Number of oranges	1	3	6		

3. How many oranges does Mrs. Chang need for pile # 6? _____
   Show how you figured it out.

4. Mrs. Chang says, "I need 44 oranges to make pile # 9 ".
   Without drawing a diagram, explain how you know that she is wrong.

_____

_____

_____

5. How many oranges does she need to make pile # 9? _____

8

Piles of Oranges	Test 4 Rubric	
The core elements of performance required by this task are: • to describe, extend and make generalizations about a number pattern Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Draws a correct diagram for pile # 4.	2	2
2. Gives correct answers **10** and **15** and writes these numbers in the table.	1, 1	2
3. Gives correct answer: **21** Shows work such as: 1 + 2 + 3 + 4 + 5 + 6 = 21 Accept adding on 6 from pile # 5: 15 + 6 **or** Draws a correct diagram to show pile # 5.	1 1 **or** 1	2
4. Gives a correct explanation such as: In pile # 9, the number of oranges is 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 45. Accept adding on from, say, pile # 6.	1	1
5. Gives correct answer: **45**	1	1
Total Points		**8**

Looking at Student Work on Piles of Oranges

Student A is able to accurately draw the piles of oranges, while maintaining the linear rows. The student sees the pattern of growing addition underneath the drawings and uses it to solve further parts of the task. The student also uses a grouping strategy to simplify the final addition problem.

Student A

Piles of Oranges

This problem gives you the chance to:
• describe, extend and make generalizations about a number pattern

Here are some piles of oranges that are displayed in Mrs. Chang's grocery store.

	pile # 1	pile # 2	pile # 3	pile # 4
number of oranges	1	1 + 2 = 3	1 + 2 + 3 = 6	1+2+3+4=10 ✓

1. Draw pile # 4 of oranges next to pile # 3 in the diagram above.

2. How many oranges are needed for pile # 4 and pile # 5?
Write your answers in the table below.

Pile #	1	2	3	4	5
Number of oranges	1	3	6	10 ✓	15 ✓

3. How many oranges does Mrs. Chang need for pile # 6?
Show how you figured it out. 21 ✓✓

1+2+3+4+5+6=21 ✓

4. Mrs. Chang says, "I need 44 oranges to make pile # 9."
Without drawing a diagram, explain how you know that she is wrong.
If you add 1+2+3+4+5+6+7+8+9 it doesn't equal 44. ✓✓

5. How many oranges does she need to make pile # 9? 45 ✓✓

9 — 9
8 +
7 > 15
6
5 > 11
4 > 7
3
2 > 3
1
9 — 45

Student B notices that the pattern is growing by adding a larger number each time. The student uses this generalization to make a table to solve for the number of oranges in pile #9.

Student B

Pile #	1	2	3	4	5
Number of oranges	1	3	6	10	15

3. How many oranges does Mrs. Chang need for pile # 6? 21
Show how you figured it out.
15
the numbers are growing by 3, 4, 5, 6... so I added a fifteen and a six with a +6 21

4. Mrs. Chang says, "I need 44 oranges to make pile # 9".
Without drawing a diagram, explain how you know that she is wrong.
she neds 45 orenges to make pile nine

pile	number of oranges
1	1
2	3
3	6
4	10
5	15
6	21
7	28
8	36
9	45
10	55

5. How many oranges does she need to make pile # 9? 45

Many students had difficulty explaining why 44 was the wrong number of oranges. Students, like Student C, often did not give enough information. Why is 44 not enough? How did you know 44 was not enough? Student D sees the addition pattern as evidenced in part 3, but doesn't use this to explain why the counting is incorrect or how he got the 45. Student E had some strategy for finding the total number of oranges and reasons about the missing one orange. All these students can't connect their logic of getting a correct solution to a justification in part 4.

Student C

4. Mrs. Chang says, "I need 44 oranges to make pile # 9".
Without drawing a diagram, explain how you know that she is wrong.
I know shes wrong because 44 oranges are not enough

5. How many oranges does she need to make pile # 9? 45 oranges

Student D

4. Mrs. Chang says, "I need 44 oranges to make pile # 9 ".
Without drawing a diagram, explain how you know that she is wrong.

I think that she is wrong because she didn't counted write.

5. How many oranges does she need to make pile # 9? 45

Student E

4. Mrs. Chang says, "I need 44 oranges to make pile # 9 ".
Without drawing a diagram, explain how you know that she is wrong.

I bet she didn't count the top 10 orange cause the answer is 45. A

5. How many oranges does she need to make pile # 9? 45

Student F is not looking for mathematical sequences or functions. The student tries to find a pattern of even and odd numbers, which is not useful for working with a growth pattern.

Student F

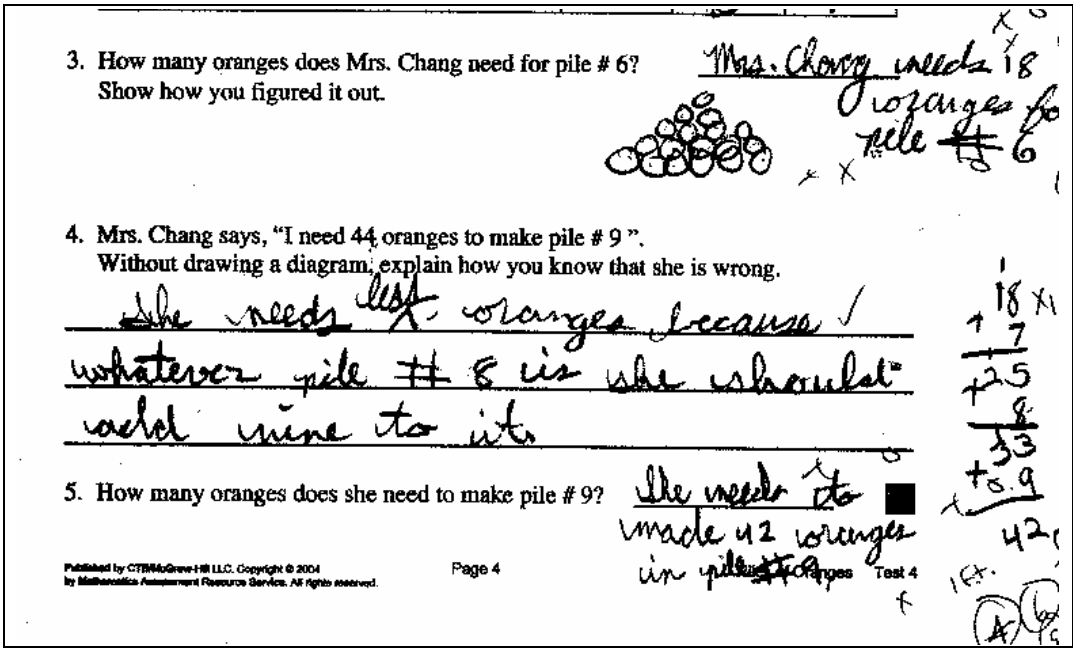
4. Mrs. Chang says. "I need 44 oranges to make pile # 9 ".
Without drawing a diagram, explain how you know that she is wrong.

Mrs. Chang is wrong becuse 4 is an even number and 9 is odd.

5. How many oranges does she need to make pile # 9? 50

Student G sees a pattern of adding an increasing larger number each time. However the student struggles with making a good mathematical representation and gets an incorrect value for 6. Students need lots of practice with making and seeing mathematical representations to develop the ability to see and draw in rows.

Student G



3. How many oranges does Mrs. Chang need for pile # 6? Show how you figured it out.

Mrs. Chang needs 18 oranges for pile #6

4. Mrs. Chang says, "I need 44 oranges to make pile # 9 ". Without drawing a diagram, explain how you know that she is wrong.

She needs less oranges because whatever pile # 8 is she should add nine to it.

18 x1
+7
+25
33
+9
42

5. How many oranges does she need to make pile # 9?

She needs to make 42 oranges in pile #9

Many students look for an easy adding on pattern in tables, rather than trying to think about how the physical model grows. Student H sees the oranges growing from 3 to 6 and decides the pattern is adding 3 every time. Almost half the students who made error on the table made this mistake.

Student H

	pile # 1	pile # 2	pile # 3	pile # 4
number of oranges	1	1 + 2 = 3	1 + 2 + 3 = 6	1+2+3+3=9

1. Draw pile # 4 of oranges next to pile # 3 in the diagram above.

2. How many oranges are needed for pile # 4 and pile # 5?
Write your answers in the table below.

Pile #	1	2	3	4	5
Number of oranges	1	3	6	9	12

3. How many oranges does Mrs. Chang need for pile # 6? 15
Show how you figured it out. I added in my head all the numbers

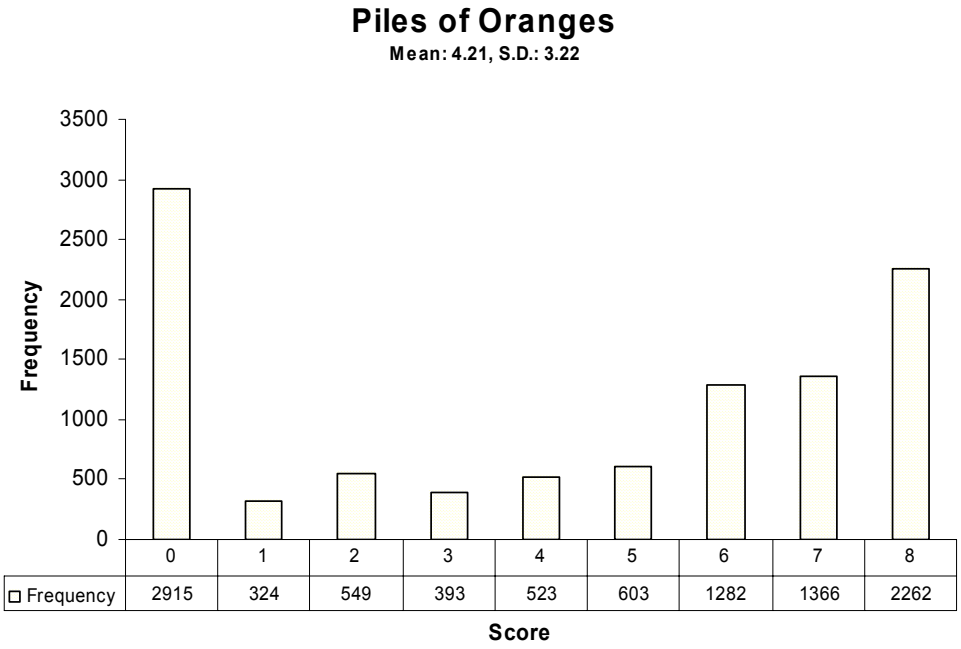
4. Mrs. Chang says, "I need 44 oranges to make pile # 9 ".
Without drawing a diagram, explain how you know that she is wrong.

She is wrong because if you add up the numbers she is off.

5. How many oranges does she need to make pile # 9? 24

Teacher's Notes:

Frequency Distribution for each Task – Grade 4
Grade 4 – Piles of Oranges



Score:	0	1	2	3	4	5	6	7	8
% < =	28.5%	31.7%	37.1%	40.9%	46.0%	51.9%	64.5%	77.9%	100.0%
% > =	100.0%	71.5%	68.3%	62.9%	59.1%	54.0%	48.1%	35.5%	22.1%

There is a maximum of 8 points available for this task.
The cut score for a level 3 response, meeting standard, is 4 points.

Many students (about 68%) could correctly draw the next pile of oranges in the sequence. More than half the students (60%) could continue the pattern in drawings or in the table. Almost half the students (48%) could continue the pattern in drawings, in the table, and to the next number beyond the table. Almost 36% of the students could extend the pattern in pictures, tables, numbers, and use a numerical pattern to work beyond the numbers in the table from 5 to 9. 22% of the students could meet all the demands of the task including making a mathematical justification for why 44 is the incorrect number of oranges for pile 9. Almost 29% of the students scored no points on this task. All the students in the survey attempted the task.

Piles of Oranges

Points	Understandings	Misunderstandings
0	All the students with this score attempted the task.	Students had difficulty drawing the pattern. Some students drew the total number of oranges in a straight row, made square boxes of oranges, or left out a row. These students were not able to think about the attributes of the physical pattern.
2	Students could draw the next figure in the sequence.	When filling out the table, many students (50%) tried to add on by 3's every time. This also led them to calculate an answer of 15 for part 3.
4	Students could draw pile #4 and fill out the table.	
6	Students could fill in the table, find the number of oranges in pile 6 and explain how they got it, and either draw pile 4 or solve for pile 9 and explain why it is not 44 oranges.	Even though the drawing is the easiest points for most students to get, it is also one of the most difficult points for other students to get. The spatial relationships required are not trivial. Students who missed question 5 were usually thinking about groups of 3. If they added on by three's from the table they would get an answer of 24 (7% of the students) or they multiplied 9 x 3 = 27 (12%).
7		Students could do the entire task except explain why 44 was incorrect. Most students were too vague, "It's not enough." "She counted wrong." "She forgot the one on the top."
8	Students could extend a pattern using pictures, tables, and patterns. They could make a justification for why a number does not fit the sequence of piles of oranges without using a drawing.	

Based on teacher observations, this is what fourth graders knew and were able to do:

- Extend pattern with drawing
- Extend pattern using a table
- Add on with the pattern to find number for pile 6 and for pile 9
- Show or explain thinking for how they extended the pattern

Areas of difficulty for fourth graders, fourth graders struggled with:

- Explaining the patterns in words
- Explaining why 44 does not fit the pattern/ analyzing the thinking of other students
- Generalizing using all available information
- Writing equalities

Questions for Reflection on Piles of Oranges

- What types of growing pattern problems have your students worked with this year?
- When looking for patterns are students asked to describe what stays the same and what changes? How does looking at the geometrical attributes help them extend the numerical pattern and make generalizations?
- What activities do students do to help them build their skills at spatial visualization and mathematical representations? When looking at arrays do you think your students see rows and columns? How many are still not at that level of development?
- When looking at patterns, are students asked to justify their thinking? What habits of mind might have helped students realize the pattern was not just growing by 3's? What clues were available to them?
- What types of opportunities do students get in your classroom to practice and develop their ability to make justifications?
- What strategies did students use to solve part 3 and part 4? Did they make a drawing? Extend the table? Write a number sentence? Other?

Teacher's Notes:

Implications for Instruction:

Students need to learn to describe how a pattern grows in words, by explaining what stays the same and what changes. Students need to look at all the values in a table to determine the pattern, not just the last two. Students who give good mathematical explanations often show calculations and explain how they picked those numbers. Students at fourth grade should be more specific than "I did it in my head" or "it didn't follow the pattern."